In this comparison, we use the same SPIHT's quantizer and entropy coder, A. Said and W. A. Pearlman, "A new fast and efficient image coder based on set partitioning in hierarchical trees," *IEEE Trans on Circuits Syst. Video Tech.*, vol. 6, pp. 243–250, June 1996, for every transform. In the block-transform cases, we use the modified zero-tree structure in T. D. Tran and T. Q. Nguyen, "A lapped transform embedded image coder," *ISCAS,* Monterey, May 1998, where each block of transform coefficients is treated analogously to a full wavelet tree and three more levels of decomposition are employed to decorrelate the DC subband further.

Table 1 contains a comparison of the complexity of these four coding systems, comparing numbers of operations needed per 8 transform coefficients:

Transform	No. Multiplications	No. Additions	No. Shifts
8 × 8 DCT	13	29	0
8 × 16 Type-I Fast LOT	22	54	0
917 Wavelet, 1-level	36	56	0
8 × 6 Fast LiftLT	14	51	6

In such a comparison, the number of multiplication operations dominates the "cost" of the transform in terms of computing resources and time, and number of additions and number of shifts have negligible effect. In this table, it is clear that the fast LiftLT is almost as low as the DCT in complexity and more than twice as efficient as the wavelet transform.

Table 2 sets forth a number of different performance measures for each of the four methods:

Transform	Coding Gain (dB)	DC Atten. (−dB)	Stopband Atten. (−dB)	Mir. Freq. Atten. (−dB)
8 × 8 DCT	8.83	310.62	9.96	322.1
8 × 16 Type-I Fast LOT	9.2	309.04	17.32	314.7
8 × 16 Optional LT	9.62	327.4	13.5	55.54
8 × 16 Fast LiftLT	9.54	312.56	13.21	304.85

The fast LiftLT is comparable to the optional 8x16 LT transform in coding gain and stopband attenuation an significantly better than the DCT.

Reconstructed images for a standard 512x512 "Barbara" test image at 1:32 compression ratio are shown in FIG. **9** for aesthetic and heuristic evaluation. Top left **21** is the reconstructed image for the 8x8 DCT (27.28 dB PSNR); top right shows the result for the 8x16 LOT(28.71 dB PSNR); bottom left is the 9/7 tap wavelet reconstruction (27.58 dB PSNR); and bottom right, 8x16 LiftLT (28.93 dB PSNR). The objective coding results for standard 512x512 "Lena," "Goldhill," and "Barbara" test image (PSNR in dB's) are tabulated in Table 3:

	Lena				Goldhill				Barbara			
Comp. Ratio	9/7 WL SPIHT	8 × 8 DCT	8 × 16 LOT	8 × 16 LiftLT	9/7 WL SPIHT	8 × 8 DCT	8 × 16 LOT	8 × 16 LiftLT	9/7 WL SPIHT	8 × 8 DCT	8 × 16 LOT	8 × 16 LiftLT
8	40.41	39.91	40.02	40.21	36.55	36.25	36.56	36.56	36.41	36.31	37.22	37.57
16	37.21	36.38	36.69	37.11	33.13	32.76	33.12	33.22	31.4	31.11	32.52	32.82
32	34.11	32.9	33.49	34	30.56	30.07	30.52	30.63	27.58	27.28	28.71	28.93
64	31.1	29.67	30.43	30.9	28.48	27.93	28.34	28.54	24.86	24.58	25.66	25.93
100	29.35	27.8	28.59	29.03	27.38	26.65	27.08	27.28	23.76	23.42	24.32	24.5
128	28.38	26.91	27.6	28.12	26.73	26.01	26.46	26.7	23.35	22.68	23.36	23.47

PSNR is an acronym for power signal to noise r-atio and represents the logarithm of the ratio of maximum amplitude squared to the mean square error of the reconstructed signal expressed in decibels (dB).

The LiftLT outperforms its block transform relatives for all test images at all bit rates. Comparing to the wavelet transform, the LiftLT is quite competitive on smooth images—about 0.2 dB below on Lena. However, for more complex images such as Goldhill or Barbara, the LiftLT consistently surpasses the 9/7-tap wavelet. The PSNR improvement can reach as high as 1.5 dB.

FIG. **9** also shows pictorially the reconstruction performance in Barbara images at 1:32 compression ratio for heuristic comparison. The visual quality of the LiftLT reconstructed image is noticeably superior. Blocking is completely avoided whereas ringing is reasonably contained. Top left: 8x8 DCT, 27.28 dB. Top right: 8x16 LOT, 28.71 dB. Bottom left: 9/7-tap wavelet, 27.58 dB. Bottom right: 8x16 LiftLT, 28.93 dB. Visual inspection indicates that the LiftLT coder gives at least as good performance as the wavelet coder. The appearance of blocking artifacts in the DCT reconstruction (upper left) is readily apparent. The LOT transform result (upper right) suffers visibly from the same artifacts even though it is lapped. In addition, it is substantially more complex and therefore slower than the DCT transform. The wavelet transform reconstruction (lower left) shows no blocking and is of generally high quality for this level of compression. It is faster than the LOT but significantly slower than the DCT. Finally, the results of the LiftLT transform are shown at lower right. Again, it shows no blocking artifacts, and the picture quality is in general comparable to that of the wavelet transform reconstruction, while its speed is very close to that of the bare DCT.

We claim:

1. An apparatus for coding, storing or transmitting, and decoding MxM sized blocks of digitally represented images, where M is an even number comprising

a. a forward transform comprising

i. a base transform having M channels numbered 0 through M−1, half of said channel numbers being odd and half being even;

ii. an equal normalization factor in each of the M channels selected to be dyadic-rational;

iii. a full-scale butterfly implemented as a series of lifting steps with a first set of dyadic rational coefficients;

iv. M/2 delay lines in the odd numbered channels;

v. a full-scale butterfly implemented as a series of lifting steps with said first set of dyadic rational coefficients; and

vi. a series of lifting steps in the odd numbered channels with a second specifically selected set of dyadic-rational coefficients;

b. means for transmission or storage of the transform output coefficients; and

c. an inverse transform comprising

i. M channels numbered 0 through M−1, half of said channel numbers being odd and half being even;

ii. a series of inverse lifting steps in the odd numbered channels with said second set of specifically selected dyadic-rational coefficients;

iii. a full-scale butterfly implemented as a series of lifting steps with said first set of specifically selected dyadic-rational coefficients;

iv. M/2 delay lines in the even numbered channels;

v. a full-scale butterfly implemented as a series of lifting steps with said first set of specifically selected dyadic-rational coefficients;

vi. an equal denormalization factor in each of the M channels specifically selected to be dyadic-rational; and

vii. a base inverse transform having M channels numbered 0 through M−1.

2. The apparatus of claim **1** in which the normalizing factor takes the value 25/16 and simultaneously the denormalizing factor takes the value 16/25.

3. The apparatus of claim **1** in which the normalizing factor takes the value 5/4 and simultaneously the denormalizing factor takes the value 4/5.

4. The apparatus of claim **1** in which the first set of dyadic rational coefficients are all equal to 1.

5. The apparatus of claim **1** in which the second set of dyadic rational coefficients are all equal to ½.

6. The apparatus of claim **1** in which the base transform is any M×M invertible matrix of the form of a linear phase filter and the inverse base transform is the inverse of said M×M invertible matrix.

7. The apparatus of claim **1** in which the base transform is the forward M×M discrete cosine transform and the inverse base transform is the inverse M×M discrete cosine transform.

8. An apparatus for coding, compressing, storing or transmitting, and decoding a block of M×M intensities from a digital image selected by an M×M window moving recursively over the image, comprising:

a. an M×M block transform comprising:

i. an initial stage

ii. a normalizing factor in each channel

b. a cascade comprising a plurality of dyadic rational lifting transforms, each of said plurality of dyadic rational lifting transforms comprising

i. a first bank of pairs of butterfly lifting steps with unitary coefficients between adjacent lines of said transform;

ii. a bank of delay lines in a first group of M/2 alternating lines;

iii. a second bank of butterfly lifting steps with unitary coefficients, and

iv. a bank of pairs of butterfly lifting steps with coefficients of 1/2 between M/2−1 pairs of said M/2 alternating lines;

c. means for transmission or storage of the output coefficients of said M×M block transform; and

d. an inverse transform comprising

i. a cascade comprising a plurality of dyadic rational lifting transforms, each of said plurality of dyadic rational lifting transforms comprising

a) a bank of pairs of butterfly lifting steps with coefficients of 1/2 between said M/2−1 pairs of said M/2 alternating lines;

b) a first bank of pairs of butterfly lifting steps with unitary coefficients between adjacent lines of said transform;

c) a bank of delay lines in a second group of M/2 alternating lines, and

d) a second bank of pairs of butterfly lifting steps with unitary coefficients between adjacent lines of said transform;

ii. a de-scaling bank, and

iii. an inverse initial stage.

9. A method of coding, storing or transmitting, and decoding M×M sized blocks of digitally represented images, where M is an even number, comprising

a. transmitting the original picture signals to a coder, which effects the steps of

i. converting the signals with a base transform having M channels numbered 0 through M−1, half of said channel numbers being odd and half being even,

ii. normalizing the output of the preceding step with a dyadic rational normalization factor in each of said M channels;

iii. processing the output of the preceding step through two lifting steps with a first set of identical dyadic rational coefficients connecting each pair of adjacent numbered channels in a butterfly configuration;

iv. transmitting the resulting coefficients through M/2 delay lines in the odd numbered channels;

v. processing the output of the preceding step through two inverse lifting steps with the first set of dyadic rational coefficients connecting each pair of adjacent numbered channels in a butterfly configuration; and

vi. applying two lifting steps with a second set of identical dyadic rational coefficients connecting each pair of adjacent odd numbered channels to the output of the preceding step;

b. transmitting or storing the transform output coefficients;

c. receiving the transform output coefficients in a decoder; and

d. processing the output coefficients in a decoder, comprising the steps of

i. receiving the coefficients in M channels numbered 0 through M−1, half of said channel numbers being odd and half being even;

ii. applying two inverse lifting steps with dyadic rational coefficients connecting each pair of adjacent odd numbered channels;

iii. applying two lifting steps with dyadic rational coefficients connecting each pair of adjacent numbered channels in a butterfly configuration;

iv. transmitting the result of the preceding step through M/2 delay lines in the even numbered channels;

v. applying two inverse lifting steps with dyadic rational coefficients connecting each pair of adjacent numbered channels in a butterfly configuration;

vi. denormalizing the result of the preceding step with a dyadic rational inverse normalization factor in each of said M channels; and

vii. processing the result of the preceding step through a base inverse transform having M channels numbered 0 through M−1.

10. A method of coding, compressing, storing or transmitting, and decoding a block of M×M intensities from a digital image selected by an M×M window moving recursively over the image, comprising the steps of:

a. Processing the intensities in an M×M block coder comprising the steps of:
   i. processing the intensities through an initial stage;
   ii. scaling the result of the preceding step in each channel;
b. processing the result of the preceding step through a cascade comprising a plurality of dyadic rational lifting transforms, each of said plurality of dyadic rational lifting transforms comprising
   i. a first bank of pairs of butterfly lifting steps with unitary coefficients between adjacent lines of said transform;
   ii. a bank of delay lines in a first group of M/2 alternating lines;
   iii. a second bank of butterfly lifting steps with unitary coefficients, and
   iv. a bank of pairs of butterfly lifting steps with coefficients of 1/2 between M/2−1 pairs of said M/2 alternating lines;
c. transmitting or storing the output coefficients of said M×M block coder;
d. receiving the output coefficients in a decoder; and
e. processing the output coefficients in the decoder, comprising the steps of
   i. processing the output coefficients through a cascade comprising a plurality of dyadic rational lifting transforms, each of said plurality of dyadic rational lifting transforms comprising
      a) a bank of pairs of butterfly lifting steps with coefficients of 1/2 between said M/2−1 pairs of said M/2 alternating lines;
      b) a first bank of pairs of butterfly lifting steps with unitary coefficients between adjacent lines of said transform;
      c) a bank of delay lines in a second group of M/2 alternating lines;
      d) a second bank of pairs of butterfly lifting steps with unitary coefficients between adjacent lines of said transform;
      e) a de-scaling bank; and
f. processing the results of the preceding step in an inverse initial stage.

11. The apparatus of claim 1 in which the constants are approximations chosen for rapid computing rather than exact constants.

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